

Correspondence between Jordan-Einstein frames and Palatini-metric formalisms

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We discuss the conformal symmetry between Jordan and Einstein frames considering their relations with the metric and Palatini formalisms for modified gravity. Appropriate conformal transformations are taken into account leading to the evident connection between the gravitational actions in the two mentioned frames and the Hilbert-Einstein action with a cosmological constant. We show that the apparent differences between Palatini and metric formalisms strictly depend on the representation while the number of degrees of freedom is preserved. This means that the dynamical content of both formalism is identical.

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I. INTRODUCTION

The recent interest in investigating alternative theories of gravity has arisen from cosmology, quantum field theory and Mach’s principle. The initial singularity, flatness and horizon problems [1] indicate that the standard cosmological model, based on general relativity (GR) and the particle standard model [2–4], fails in describing the Universe at extreme regimes. Besides, GR does not work as a fundamental theory capable of giving a quantum description of spacetime [5]. For these reasons and due to the lack of a definitive quantum gravity theory, alternative theories of gravitation have been pursued in order to attempt an at least semi-classical approach to quantization and to early universe shortcomings.

In particular extended theories of gravity (ETGs) [6–8] take into account the problem of gravitational interaction correcting and enlarging the Einstein theory by the introduction of non-minimally coupled scalar fields and higher-order terms in curvature invariants. The idea to extend GR is fruitful and economic with respect to several attempts which try to solve problems by adding new and, sometime, unjustified new ingredients in order to give a self-consistent picture of the cosmic and quantum dynamics (e.g. dark energy and dark matter up to now not detected at fundamental level). In particular, such an approach ‘naturally’ reproduce inflationary behaviors in early epochs and is capable of matching with several astrophysical observations. Besides, the present-day observed accelerated expansion of Hubble flow and the missing matter of astrophysical large-scale structures could be explained by changing the gravitational sector, i.e. the lhs of the field equations [9]. The alternative philosophy is to add new cosmic fluids (new components in the rhs of the field equations) which should give rise to clustered structures (dark matter) or to accelerated dynamics (dark energy) thanks to exotic equations of state. In particular, relaxing the hypothesis that gravitational Lagrangian has to be a linear function of the Ricci cur-

vature scalar R , as in the Hilbert-Einstein formulation, one can take into account, as a minimal extension, an effective action where the gravitational Lagrangian is a generic $f(R)$ function [10–15].

Moreover one can consider actions where scalar field are non-minimally coupled to gravity [16] as generalization of the Brans-Dicke theory [17]. Through the conformal transformations, it is possible to show that any higher-order or scalar-tensor theory, in absence of ordinary matter, e.g. a perfect fluid, is conformally equivalent to an Einstein theory plus minimally coupled scalar fields. In principle, the converse is also true: we can transform standard Einstein gravity plus minimally coupled scalar fields into a non-minimally coupled scalar-tensor theory.

Conformal transformations can be useful to point out common features between Palatini and metric approaches to gravitational interaction. The fundamental idea of the Palatini formalism is to consider the connection Γ , entering the definition of the Ricci tensor, to be independent of the metric g defined on the spacetime manifold \mathcal{M} . Conceptually, this means that geodesic and causal structures on \mathcal{M} can be disentangled [18]. The Palatini formulation for the standard Hilbert-Einstein theory results to be equivalent to the purely metric theory: this follows from the fact that the field equations for the connection Γ , firstly considered to be independent of the metric, give the Levi-Civita connection of the metric g . As a consequence, there is no reason to impose the Palatini variational principle in the standard Hilbert-Einstein theory instead of the metric variational principle. The situation changes if we consider ETGs, depending on functions of curvature invariants, as $f(R)$, or non-minimally coupled to some scalar field. In these cases, the Palatini and the metric variational principles provide different field equations and the theories thus derived seem to differ [19, 20]. This status of art is not comfortable since dynamics and its predictions should not depend on the representation. In fact, it is well known that several astrophysical and

cosmological observations can be well interpreted in a formalism and not in the other and viceversa [6, 7]. This shortcoming can be partially removed by investigating how Palatini and metric formalisms are related by conformal transformations.

In this paper, we discuss the correspondence between Jordan-Einstein frames and Palatini-metric formalisms pointing out how Lagrangians can be transformed between each other and that the number of degrees of freedom is preserved.

In Sec.II we discuss the conformal symmetry between Jordan and Einstein frames. In Sec.III we introduce metric and Palatini formalisms for some ETGs, and in Sec.IV, we use some appropriate transformations from Jordan to Einstein frames in view to compare Palatini and metric formalisms. Conclusions and some physical considerations are given in Sec.V.

II. CONFORMAL SYMMETRY BETWEEN JORDAN AND EINSTEIN FRAMES

The general form of the action in four dimensions when there is a nonstandard coupling between a scalar field and the geometry is

$$\mathcal{S} = \int d^4x \sqrt{-g} \left(F(\phi)R + \frac{1}{2}g^{\mu\nu}\phi_{;\mu}\phi_{;\nu} - V(\phi) \right), \quad (1)$$

where R is the Ricci scalar, $V(\phi)$ and $F(\phi)$ are functions describing the effective potential and the coupling of ϕ with gravity, respectively¹. This form of the action or the related Lagrangian density is usually referred to the *Jordan frame*. The variation with respect to the metric $g_{\mu\nu}$ gives the generalized Einstein equations

$$F(\phi)G_{\mu\nu} = -\frac{1}{2}T_{\mu\nu} - g_{\mu\nu}\square_\Gamma F(\phi) + F(\phi)_{;\mu\nu}, \quad (2)$$

where \square_Γ is the d'Alembert operator with respect to the connection Γ , $G_{\mu\nu}$ is the standard Einstein tensor

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}, \quad (3)$$

and $T_{\mu\nu}$ is the energy-momentum tensor of the scalar field

$$T_{\mu\nu} = \phi_{;\mu}\phi_{;\nu} - \frac{1}{2}g_{\mu\nu}\phi_{;\alpha}\phi^{;\alpha} + g_{\mu\nu}V(\phi). \quad (4)$$

The variation with respect to ϕ leads to the Klein-Gordon equation

$$\square_\Gamma \phi - RF_\phi(\phi) + V_\phi(\phi) = 0, \quad (5)$$

where $F_\phi = \frac{dF(\phi)}{d\phi}$, $V_\phi(\phi) = \frac{dV(\phi)}{d\phi}$. Let us consider now a conformal transformation on the metric $g_{\mu\nu}$

$$\bar{g}_{\mu\nu} = e^{2\omega} g_{\mu\nu}, \quad (6)$$

with the conformal factor $e^{2\omega}$. The Lagrangian density in (1) becomes

$$\sqrt{-g} \left(FR + \frac{1}{2}g^{\mu\nu}\phi_{;\mu}\phi_{;\nu} - V(\phi) \right) = \sqrt{-\bar{g}}e^{-2\omega} \left(F\bar{R} - 6F\square_{\bar{\Gamma}}\omega - 6F\omega_{;\alpha}\omega^{;\alpha} + \frac{1}{2}\bar{g}^{\mu\nu}\phi_{;\mu}\phi_{;\nu} - e^{-2\omega}V \right), \quad (7)$$

where \bar{R} , $\bar{\Gamma}$ and $\square_{\bar{\Gamma}}$ are the corresponding quantities with respect to the metric $\bar{g}_{\mu\nu}$ and connection $\bar{\Gamma}$, respectively. If we require that the new Lagrangian, in terms of $\bar{g}_{\mu\nu}$, appears as a standard Einstein theory, the conformal factor has to be related to F as

$$e^{2\omega} = 2F. \quad (8)$$

Using this relation, the Lagrangian (7) becomes

$$\sqrt{-g} \left(FR + \frac{1}{2}g^{\mu\nu}\phi_{;\mu}\phi_{;\nu} - V(\phi) \right) = \sqrt{-\bar{g}} \left(\frac{1}{2}\bar{R} + 3\square_{\bar{\Gamma}}\omega + \frac{3F_\phi^2 - F}{4F^2}\phi_{;\alpha}\phi^{;\alpha} - \frac{V}{4F^2} \right), \quad (9)$$

By introducing a new scalar field $\bar{\phi}$ and the related potential \bar{V} defined as

$$\bar{\phi}_{;\alpha} = \sqrt{\frac{3F_\phi^2 - F}{4F^2}}\phi_{;\alpha}, \quad \bar{V}(\bar{\phi}(\phi)) = \frac{V}{4F^2}, \quad (10)$$

we obtain²

$$\sqrt{-g} \left(FR + \frac{1}{2}g^{\mu\nu}\phi_{;\mu}\phi_{;\nu} - V(\phi) \right) = \sqrt{-\bar{g}} \left(\frac{1}{2}\bar{R} + \frac{1}{2}\bar{\phi}_{;\alpha}\bar{\phi}^{;\alpha} - \bar{V} \right), \quad (11)$$

where the r.h.s. is the usual Einstein-Hilbert Lagrangian density subject to the metric $\bar{g}_{\mu\nu}$, plus the standard Lagrangian density of the scalar field $\bar{\phi}$. This form of the Lagrangian density is usually referred to the *Einstein frame*. Therefore, we realize that any non-minimally coupled theory of gravity with scalar field, in absence of ordinary matter, is conformally equivalent to the standard Einstein gravity coupled with scalar field provided we use the conformal transformation (8) together with the definitions (10). The converse is also true: for a given $F(\phi)$, such that

$$3F_\phi^2 - F > 0, \quad (12)$$

¹ The metric signature is $(-+++)$ and Planck units are adopted.

² Note that the divergence-type term $3\square_{\bar{\Gamma}}\omega$ appearing in the Lagrangian density is not considered [22].

that means the Hessian determinant is non singular and the coupling has the right signature, we can transform a standard Einstein theory into a nonstandard coupled theory. This has an important meaning: if we are able to solve the field equations within the framework of standard Einstein gravity coupled with a scalar field subject to a given potential, we are able, in principle, to get solutions for the class of nonstandard coupled theories, with the coupling $F(\phi)$, through the conformal transformation defined by (8), the only constraint being the second equation of (10). This statement is exactly what we mean as the *conformal equivalence between Jordan and Einstein frames*. However, this mathematical equivalence does not imply directly the physical equivalence of the two frames. Examples in this sense can be found in [23–25].

III. METRIC AND PALATINI FORMALISM FOR MODIFIED GRAVITY

The action in the metric formalism for $f(R)$ gravity takes the form

$$\mathcal{S} = \int_m d^4x \sqrt{-g} f(R). \quad (13)$$

In the metric formalism, the variation of the action is accomplished with respect to the metric. One can show that this action dynamically corresponds to an action of non-minimally coupled gravity with a new scalar field having no kinetic term. By introducing a new auxiliary field χ , the dynamically equivalent action can be rewritten as [7, 21]

$$\mathcal{S} = \int_m d^4x \sqrt{-g} (f(\chi) + f'(\chi)(R - \chi)). \quad (14)$$

Variation with respect to χ yields the equation

$$f''(\chi)(R - \chi) = 0. \quad (15)$$

Therefore, $\chi = R$, if $f''(\chi) \neq 0$, reproduces the action (13). Redefining the field χ by $\phi = f'(\chi)$ and introducing the potential

$$V(\phi) = \chi(\phi)\phi - f(\chi(\phi)), \quad (16)$$

the action (14) takes the form

$$\mathcal{S} = \int_m d^4x \sqrt{-g} (\phi R - V(\phi)), \quad (17)$$

that is the Jordan frame representation of the action of a Brans-Dicke theory with Brans-Dicke parameter $\omega_0 = 0$, known as O'Hanlon action in metric formalism.

Beside the metric formalism in which the variation of the action is done with respect to the metric, the Einstein equations can be derived as well using the Palatini formalism, i.e. the variation with respect to the metric is independent of the variation with respect to the connection.

The Riemann tensor and the Ricci tensor are also constructed with the independent connection and the metric is not needed to obtain the latter from the former. So, in order to make a difference with metric formalism, we shall use $\mathcal{R}_{\mu\nu}$ and \mathcal{R} instead of $R_{\mu\nu}$ and R , respectively. In the standard Einstein-Hilbert action there is no specific difference between these two formalisms. However, once we generalize the action to depend on a generalized form of the Ricci scalar they are no longer the same.

We briefly review the $f(\mathcal{R})$ gravity in Palatini formalism and show how it corresponds to a Brans-Dicke theory [6, 7]. The action in the Palatini formalism with no matter is written as

$$\mathcal{S} = \int_p d^4x \sqrt{-g} f(\mathcal{R}). \quad (18)$$

Varying the action (18) independently with respect to the metric and the connection, respectively, and using the formula

$$\delta \mathcal{R}_{\mu\nu} = \bar{\nabla}_\lambda \delta \Gamma_{\mu\nu}^\lambda - \bar{\nabla}_\nu \delta \Gamma_{\mu\lambda}^\lambda, \quad (19)$$

yields

$$f'(\mathcal{R}) \mathcal{R}_{(\mu\nu)} - \frac{1}{2} f(\mathcal{R}) g_{\mu\nu} = 0, \quad (20)$$

$$\bar{\nabla}_\lambda (\sqrt{-g} f'(\mathcal{R}) g^{\mu\nu}) - \bar{\nabla}_\sigma (\sqrt{-g} f'(\mathcal{R}) g^{\sigma(\mu} \delta^{\nu)}) = 0, \quad (21)$$

where $\bar{\nabla}$ denotes the covariant derivative defined with the independent connection $\Gamma_{\mu\nu}^\lambda$ and $(\mu\nu)$ denotes symmetrization over the indices μ, ν . Taking the trace of Eq. (21) gives

$$\bar{\nabla}_\sigma (\sqrt{-g} f'(\mathcal{R}) g^{\sigma\mu}) = 0, \quad (22)$$

by which the field equation (21) becomes

$$\bar{\nabla}_\lambda (\sqrt{-g} f'(\mathcal{R}) g^{\mu\nu}) = 0. \quad (23)$$

One may obtain some useful manipulations of the field equations. Taking the trace of Eq. (20) yields an algebraic equation for \mathcal{R}

$$f'(\mathcal{R}) \mathcal{R} - 2f(\mathcal{R}) = 0. \quad (24)$$

One can define a metric conformal to $g_{\mu\nu}$ as

$$h_{\mu\nu} = f'(\mathcal{R}) g_{\mu\nu}, \quad (25)$$

for which it is easily obtained that

$$\sqrt{-h} h^{\mu\nu} = \sqrt{-g} f'(\mathcal{R}) g^{\mu\nu}. \quad (26)$$

Eq. (23) is then the compatibility condition of the metric $h_{\mu\nu}$ with the connection $\Gamma_{\mu\nu}^\lambda$ and can be solved algebraically to give the Levi-Civita connection

$$\Gamma_{\mu\nu}^\lambda = h^{\lambda\sigma} (\partial_\mu h_{\nu\sigma} + \partial_\nu h_{\mu\sigma} - \partial_\sigma h_{\mu\nu}). \quad (27)$$

Under conformal transformation (25), the Ricci tensor and its contracted form with $g^{\mu\nu}$ become, respectively,

$$\mathcal{R}_{\mu\nu} = R_{\mu\nu} + \frac{3}{2} \frac{1}{(f'(\mathcal{R}))^2} (\nabla_\mu f'(\mathcal{R})) (\nabla_\nu f'(\mathcal{R})) + \frac{1}{(f'(\mathcal{R}))} (\nabla_\mu \nabla_\nu - \frac{1}{2} g_{\mu\nu} \square) f'(\mathcal{R}), \quad (28)$$

$$\mathcal{R} = R + \frac{3}{2} \frac{1}{(f'(\mathcal{R}))^2} (\nabla_\mu f'(\mathcal{R})) (\nabla^\mu f'(\mathcal{R})) + \frac{3}{(f'(\mathcal{R}))} \square f'(\mathcal{R}). \quad (29)$$

Note the difference between \mathcal{R} and the Ricci scalar of $h_{\mu\nu}$ is due to the fact that $g^{\mu\nu}$ is used here for the contraction of $\mathcal{R}_{\mu\nu}$. Now, by introducing a new auxiliary field χ , the dynamically equivalent action is rewritten as [7, 21]

$$\mathcal{S} = \int_P d^4x \sqrt{-g} (f(\chi) + f'(\chi)(\mathcal{R} - \chi)). \quad (30)$$

Variation with respect to χ yields the equation

$$f''(\chi)(\mathcal{R} - \chi) = 0. \quad (31)$$

Redefining the field χ by $\phi = f'(\chi)$ and introducing

$$V(\phi) = \chi(\phi)\phi - f(\chi(\phi)), \quad (32)$$

with the same request made in the metric formalism, $f''(\chi) \neq 0$ which implies $\mathcal{R} = \chi$, the action (30) takes the form

$$\mathcal{S} = \int_P d^4x \sqrt{-g} (\phi \mathcal{R} - V(\phi)). \quad (33)$$

Now, we may use $\phi = f'(\chi)$ in Eq. (29) to write down \mathcal{R} in terms of R in the action (33). This leads, modulo a surface term, to

$$\mathcal{S} = \int_P d^4x \sqrt{-g} \left(\phi R + \frac{3}{2\phi} \nabla_\mu \phi \nabla^\mu \phi - V(\phi) \right). \quad (34)$$

This is the action in Palatini formalism which corresponds to a Brans-Dicke theory with $\omega = -\frac{3}{2}$. These results are well known. How aim is now to show that the dynamical information in both metric and Palatini formalisms is the same and that the number of degrees of freedom is preserved.

IV. TRANSFORMATION FROM JORDAN TO EINSTEIN FRAMES

Let us now use some appropriate transformations to manipulate the actions (17) and (34), respectively in metric and Palatini formalisms, from the Jordan to the Einstein frame. Comparison of the action (17) with (34) reveals, as we have already specified, that the former is the action of a Brans-Dicke theory with $\omega_0 = 0$. We first define the conformal metric $\bar{g}_{\mu\nu} = \Phi g_{\mu\nu}$ and perform a conformal transformation along with $\Phi = R$ assuming

the scalar field definition $\Phi = \exp(\frac{\sqrt{3}}{2}\varphi)$. One therefore obtains an action describing Einstein gravity minimally coupled to a scalar field, that is [26, 27]

$$\mathcal{S} = \int_m d^4x \sqrt{-\bar{g}} \left(\bar{R} - \frac{1}{2} \nabla_\mu \varphi \nabla^\mu \varphi - V(\varphi) \right), \quad (35)$$

where \bar{R} is the Ricci scalar of the metric \bar{g} . This action is now said to be written in the Einstein frame.

On the other hand, if we redefine the scalar field ϕ as the new field

$$\sigma = 2\sqrt{3}\phi, \quad (36)$$

the Brans-Dicke action (34) then becomes

$$\mathcal{S} = \int_P d^4x \sqrt{-g} \left(F(\sigma)R + \frac{1}{2} g^{\mu\nu} \sigma_{;\mu} \sigma_{;\nu} - V(\sigma) \right), \quad (37)$$

where

$$F(\sigma) = \frac{1}{12} \sigma^2. \quad (38)$$

This action is now exactly the same as (1) in the Jordan frame in which ϕ is replaced by σ . However, it is worth noticing that action (37) is derived from the Palatini formalism while (1) is defined in the metric formalism. Therefore, with a similar procedure for the field σ we can write

$$\sqrt{-g} \left(F(\sigma)R + \frac{1}{2} g^{\mu\nu} \sigma_{;\mu} \sigma_{;\nu} - V(\sigma) \right) = \sqrt{-\bar{g}} \left(\frac{1}{2} \bar{R} + \frac{1}{2} \bar{\sigma}_{;\alpha} \bar{\sigma}^{;\alpha} - \bar{V} \right) \quad (39)$$

where

$$\bar{\sigma}_{;\alpha} = \sqrt{\frac{3F_\sigma^2 - F}{4F^2}} \sigma_{;\alpha}, \quad \bar{V}(\bar{\sigma}(\sigma)) = \frac{V}{4F^2}. \quad (40)$$

and

$$F_\sigma = \frac{dF(\sigma)}{d\sigma}. \quad (41)$$

Substituting $F(\sigma)$ in the definition of $\bar{\sigma}_{;\alpha}$ leads to zero kinetic term for this field and finally we obtain

$$\sqrt{-g} \left(F(\sigma)R + \frac{1}{2} g^{\mu\nu} \sigma_{;\mu} \sigma_{;\nu} - V(\sigma) \right) = \sqrt{-\bar{g}} \left(\frac{1}{2} \bar{R} - \bar{V} \right). \quad (42)$$

The r.h.s. of Eq. (42) is the Lagrangian density in the Einstein frame. It is interesting to stress that for the potential

$$V(\sigma) = \frac{\bar{\Lambda}}{36} \sigma^4, \quad (43)$$

where $\bar{\Lambda}$ is a constant, we obtain $\bar{V} = \bar{\Lambda}$ and the action in Einstein frame is reduced exactly to the Hilbert-Einstein

action with a cosmological constant $\bar{\Lambda}$. The corresponding potential in the Jordan frame with Brans-Dicke action (34) is

$$V(\phi) = 4\bar{\Lambda}\phi^2, \quad (44)$$

which converts the action into a gravity theory non-minimally coupled with a massive scalar field with an squared mass scale of the order of cosmological constant.

V. DISCUSSIONS AND CONCLUSIONS

Summarizing, we have considered four actions: metric-Jordan (17), Palatini-Jordan (34), metric-Einstein (35) and Palatini-Einstein (42). Jordan and Einstein frames, i.e. the actions (17) and (35), are related by a conformal symmetry. In this case, the appearance of a kinetic term is the relevant feature. The actions (34) and (42) are also related by a conformal symmetry. However, in this case, the kinetic term is not present. In other words, the conformal symmetry between Jordan and Einstein frames in metric and Palatini formalisms corresponds to the appearance or the vanishing of a kinetic term. On the other hand, comparing (17) with (34) reveals that the transition from metric-Jordan action (17) to Palatini-Jordan action (34) requires the appearance of a kinetic term, while the transition from metric-Einstein action (35) to the Palatini-Einstein action (42) requires the vanishing of kinetic term. This fact could have a deep dynamical interpretation. We have already learned about the conformal transformations relating Jordan with Einstein frames and Palatini with metric formalisms. Jordan and Einstein frames are *dynamically* equivalent from the conformal symmetry viewpoint. Although the metric and Palatini formalisms are connected through a conformal transformation (25), they apparently do not seem to be dynamically equivalent. Metric-Jordan action differs from Palatini-Jordan action with a dynamical *advanced* kinetic term. In the same way, metric-Einstein action differs from Palatini-Einstein action with a dynamical *retarded* kinetic term. However, the Palatini-Jordan action, when reduced to the Palatini-Einstein action, takes the same form as the metric-Jordan action, namely it becomes of the O'Hanlon type action where dynamics is completely endowed by the self-interacting potential. On the other hand, metric-Einstein action and Palatini-Jordan action represent the same dynamical features because both have a dynamical kinetic term plus a potential. In conclusion, for each map between Jordan and Einstein frames, there exists a corresponding map between Palatini and metric

formalisms. In the same way, for each map connecting two O'Hanlon type actions, namely metric-Jordan and Palatini-Einstein action, there exists a map which connects Palatini-Jordan action with metric-Einstein action. In conclusion, the dynamical content of Palatini and metric formalism is exactly the same.

Beside the mathematical consistency of Einstein relativity versus more general theories, it is important to point out the physical motivations of these approaches. In general, scalar fields are introduced to solve the shortcomings of the Standard Cosmological Model (addressed by the inflationary paradigm [28]) or issues as dark matter and dark energy (addressed by quintessence models, induced-matter theory, etc. [29]). Several results point out that a scalar field should come from a Kaluza-Klein theory than a 4D theory, and the Brans-Dicke theory could appear obsolete in this picture. For example Coley et al. have proved that all results of 4D Brans-Dicke theory can be obtained more easily from a 5D Kaluza-Klein theory (see e.g. [30, 31]) while in [32, 33] it is proved that extending General Relativity in 5D can easily give rise to mechanisms capable of generating inflation and dark energy behavior.

Beside these fundamental physics motivations, scalar fields represent the further degrees of freedom that gravitational interaction can present once we do not strictly consider General Relativity as the only possible theory of gravity. In fact, relaxing the hypothesis that the gravitational action is only the Hilbert-Einstein one, it is widely recognized that $f(R)$ -theories or theories constructed by other curvature invariants could address inflation, dark energy and dark matter problems [6–8, 10–13]. The fact that Jordan-Einstein frames and Palatini-metric formalisms have the "same" dynamical content means that the "scalar field" can be represented in several ways. However an open question remains: is it a genuine new ingredient at fundamental level (e.g. the Higgs Boson or a Kaluza-Klein field) or is it an average effect induced by geometry? Very likely the forthcoming experimental results at LHC (CERN) could give hints to address this issue.

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- [1] Guth A, *Phys. Rev. D* **23** 347 (1981).
 - [2] Wald R. M., *General Relativity*, (Chicago and London: The University of Chicago Press) (1984).
 - [3] Weinberg S., *Gravitation and Cosmology* (New York: Wi-

ley) (1972).

- [4] Misner C. W., Thorne K. S. and Wheeler J. A., *Gravitation* (San Francisco: Freeman W. H. and Company) (1970).

- [5] Birrel N.D. and Davies P.C.W., *Quantum Fields In Curved Space*, Cambridge Univ. Press, Cambridge (1982).
- [6] Capozziello S. and M. Francaviglia M., *Gen. Rel. Grav., Special Issue on Dark Energy*, **40**, 357 (2008).
- [7] Faraoni V. and Sotiriou T. P., *Rev. Mod. Phys.*, **82**, 451 (2010).
- [8] Capozziello S., De Laurentis M., *Invariance Principles and Extended Gravity: Theory and Probes*, Nova Science Publishers, New York (2010).
- [9] S. Capozziello, V.F. Cardone, A. Troisi, *JCAP*, 08, 001 (2006).
- [10] S. Capozziello, *Int. Jou. Mod. Phys. D* **11**, 483 (2002).
- [11] S. Nojiri, S.D. Odintsov, *Phys. Lett. B* **576**, 5, (2003).
- [12] S. Nojiri, S.D. Odintsov, *Phys. Rev. D* **68**, 12352 (2003).
- [13] G. Allemandi, A. Borowiec, M. Francaviglia, *Phys. Rev. D* **70**, 103503 (2004).
- [14] Vollick, D.N., *Phys. Rev. D*, 063510 (2003).
- [15] Li, B., Chu, M.C., *Phys. Rev. D* **74**, 104010 (2006).
- [16] Capozziello S., de Ritis R., Rubano C., Scudellaro P., *La Rivista del Nuovo Cimento*, 19, N4, 1 (1996).
- [17] Brans C. and Dicke R.H., *Phys. Rev.*, 124, 925 (1961) .
- [18] S. Capozziello, M.F. De Laurentis, M. Francaviglia, S. Mercadante, *Found. of Physics* **39**, 1161 (2009).
- [19] Magnano, G., Sokołowski, L.M., *Phys. Rev. D* **50**, 5039 (1994).
- [20] Ferraris, M., Francaviglia, M., Volovich, I., *Class. Quantum Grav.* **11**, 1505 (1994).
- [21] Sotiriou T. P., *Class. Quantum Grav.* **26**:152001 (2009).
- [22] S. Capozziello, R. de Ritis, A.A. Marino, *Class. Quant. Grav.* **14**, 3243 (1997).
- [23] S. Capozziello, P. Martin-Moruno, C. Rubano, *Phys. Lett.* **689 B**, 117 (2010).
- [24] S. Capozziello, S. Nojiri, S.D. Odintsov, A. Troisi, *Phys. Lett.* **639 B**, 135 (2006).
- [25] S. Capozziello, S. Nojiri, S. D. Odintsov, *Phys. Lett. B* **634**, 93 (2006).
- [26] V. Faraoni, *Cosmology in Scalar-Tensor Gravity* (Kluwer Academic Publishers, The Netherlands) (2004).
- [27] B. Whitt, *Phys. Lett. B* **575**, 176.
- [28] A. Linde *Particle Physics and Inflationary Cosmology* Harwood, Chur, Switzerland (1990).
- [29] E. J. Copeland, M. Sami, S. Tsujikawa, *Int. J. Mod. Phys. D* **15**, 1753 (2006).
- [30] A.P. Billyard, A. A. Coley, *Phys. Rev. D* **61**, 083503 (2000).
- [31] A.A. Coley, R.J. van den Hoogen, *Phys. Rev. D* **62** 023517 (2000).
- [32] J. E. Madriz Aguilar, M. Bellini, *Phys. Lett. B* **679**, 306 (2009).
- [33] J. Martin Romero, M. Bellini, *Phys. Lett. B* **674**, 143 (2009).